

***Stochastic cooling of high energy bunched beams***

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# Stochastic Cooling of High-Energy Bunched Beams\*

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## Abstract

Stochastic cooling of 100 GeV/nucleon bunched beams has been achieved in the Relativistic Heavy Ion Collider (RHIC). The physics and technology of the longitudinal cooling system are discussed, and plans for a transverse cooling system are outlined.

## INTRODUCTION

In principle, a stochastic cooling system is a wide band feedback loop[1, 2]. With system bandwidth  $W$  one obtains a time resolution  $\tau \sim 1/2W$ . For a beam of particles with charge  $q$  and current  $I$ , a longitudinal cooling system measures the average energy of samples containing  $N_s = I\tau/q$  particles each turn. This signal is filtered, amplified and applied to the beam so as to reduce the energy spread. If the beam requires  $M$  turns to mix the samples into statistical independence, the optimal cooling time scales as  $\sigma_E/\dot{\sigma}_E \approx 2N_s T_0 M$  where the revolution period is  $T_0 = 12.8\mu\text{s}$  for RHIC. Transverse pickups and kickers are used to reduce the transverse emittance and systems of both types are essential in the operation of existing antiproton sources and several low energy ion rings [3, 4, 5]. For these systems the beams are essentially, if not totally, unbunched and wide band pickup/kicker pairs work well.

A theory of bunched beam cooling was developed in the early eighties [6, 7, 8] and stochastic cooling systems for the SPS [9, 10] and the Tevatron [9, 11] were explored. Early on [11, 12, 13, 14, 15] it was found that "RF activity" extending up to very high frequencies swamped the true Schottky signal. Cooling for heavy ions in RHIC [16, 17, 18] was also considered. In RHIC, the particle densities for heavy ions are significantly lower than in the Tevatron and SPS. This, along with technological improvements, made cooling feasible in RHIC.

## THE RHIC COOLING SYSTEM

For RHIC the main purpose of the cooling system is to counteract intrabeam scattering (IBS) and keep the beam in the RF buckets. To keep costs down the signal between the pickup and the kicker travels within the tunnel in the direction opposite the beam. For a fiber optic transmission line this limited us to a delay  $2/3$  of a turn or  $T_d = 8.5\mu\text{s}$  between pickup and kicker. At this point we have worked on the yellow (counterclockwise) ring. The pickup is in the 12 o'clock straight section and the kicker is in the 4 o'clock straight section. We plan to cool at energies well above

transition so all of the phase slip is generated in the arcs and the effective delay is very close to  $2/3$  of a turn. For gold with  $\gamma = 107$  and 4 MV at  $h = 2520$  the frequency spread at the edge of the bucket is  $(\omega - \omega_0)/\omega_0 = \pm 2.8 \times 10^{-6}$ . For a one turn filter cooling system the transfer function is  $G_1(f) = [1 - \exp(i2\pi\Delta f T_0)] \exp(i2\pi\Delta f T_d)$  with  $\Delta f$  the difference between the drive frequency and the nearest revolution line. The imaginary part of  $G_1(f)$  is antisymmetric about a revolution line as is needed for cooling. The gain is correct as long as  $|\Delta f| \leq 16.5$  kHz. With the frequency spread in RHIC this limits a one turn delay cooling system to an upper frequency of 5.9 GHz. Now consider  $G_2(f) = G_1(f)[1 - \exp(i2\pi\Delta f T_0)]$ , which is two one turn delay notch filters in series. With this filter the gain has the right sign for  $|\Delta f| \leq 23.4$  kHz corresponding to an upper frequency of 8.3 GHz. The RHIC design used  $G_2$  and the upper frequency of the cooling system is 8 GHz. The lower frequency is 5 GHz.

To generate the necessary voltage note that the central part of the bunch is only 5 ns long while the bunch spacing is 106 ns. By using cavity kickers with resonant frequencies 5, 5.2, ..., 7.8, 8.0 GHz one can use Fourier decomposition to obtain the correct voltage at each bunch passage [5, 12]. A full width half power bandwidth of 10 MHz allows the cavities to change amplitude and phase between bunch passages. For a cavity with  $R/Q = 100\Omega$ , 40 Watts of amplifier power yields an rms voltage of 1.6 kV at 6.5 GHz. Both simulations and order of magnitude calculations show this is an acceptable voltage. To drive the cavities we use a traversal filter in series with  $G_2$ . Taking a delay between the filter branches of 5 ns and using 16 branches one obtains a piecewise periodic drive signal. Additional filters of 100 MHz bandwidth remove unwanted frequencies. To stop saturation the traversal filter is applied in the tunnel before the fiber-optic transmitter.

The  $\text{TM}_{0,1,0}$  mode cutoff radius at 8 GHz is 1.4 cm and we took a pipe radius of 1 cm for the cavities. To reduce aperture limitations during injection and acceleration the kicker cavities are split along the beam axis and are closed only after reaching flattop. The tanks and motors were supplied by FNAL and retrofitted for our application. There are a total of 16 cavities covering the 5 – 8 GHz band.

The gains and phases of the individual cavity drives are updated periodically during the store to track slow drifts in the optical signal path length and changes in the eigenfrequencies of the resonant cavities. This is done by first measuring the open loop system transfer function  $S(f)$ . A target transfer function  $S_0(f)$  is stored in the memory of the network analyzer. The optimal gain adjustment is ob-

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tained by minimizing

$$\int df |S(f) - GS_0(f)|^2,$$

with respect to the complex number  $G$ . The system loops through all the cavities. The one turn delay filters also undergo periodic adjustment. This is done by using the network analyzer to modulate a Mach-Zender interferometer inserted in the optical path, and adjusting the minimum of the notch frequency via computer controlled optical trombones. More details of the system as well as the results of an experiment using a low energy proton bunch can be found in [21, 22]

## RHIC DATA AND COMPARISON WITH SIMULATIONS

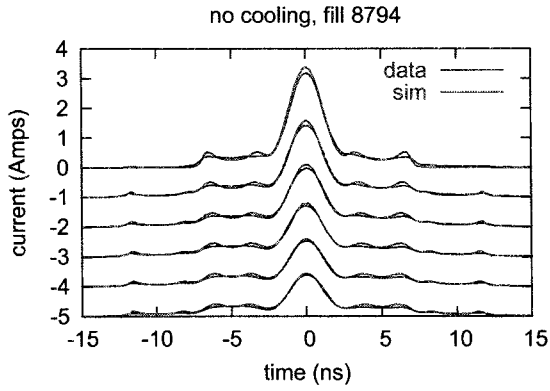


Figure 1: Evolution of the average bunch profile over a five hour RHIC store with gold beam and no cooling. The red lines are wall current monitor data and the blue lines are from a simulation. The initial conditions are shown at the top and the traces are one hour apart.

The time evolution of uncooled bunches and a simulation of them are shown in figure 1. The simulation used a simple kick code for the single particle dynamics. Relevant RHIC parameters are shown in Table 1. The effect of IBS was included by first calculating the rms growth rates

parameter	value
h=360 voltage	300 kV
h=2420 voltage	3 MV
initial FWHM bunch length	3 ns
particles/bunch	$10^9$
initial emittance	$15\pi\mu\text{m}$
betatron tunes	$Q_x = 28.2, Q_y = 27.2$
Lorentz factor	107
circumference	3834 m
transition gamma	22.89

Table 1: Machine and Beam Parameters for Gold

for the actual beam being simulated. This was done using Piwinski's formula [23] with the smooth lattice approximation. The amplitude growth rates are

$$\frac{1}{\sigma_x^2} \frac{d\sigma_x^2}{dt} = \alpha_{x0}, \quad (1)$$

$$\frac{1}{\sigma_y^2} \frac{d\sigma_y^2}{dt} = \alpha_{y0}, \quad (2)$$

$$\frac{1}{\sigma_p^2} \frac{d\sigma_p^2}{dt} = \alpha_{p0}. \quad (3)$$

For the actual RHIC beam one finds comparable growth in the two transverse directions,  $\alpha_x \approx \alpha_y$ , so the next step is to define an average transverse growth rate for the physical beam  $\alpha_{\perp 0} = (\alpha_{x0} + \alpha_{y0})/2$ . Typical rms growth times are of order an hour, but there is no need to directly simulate such a large number of turns. Instead, one can simply choose the number of simulation turns one wishes to calculate in order to model a given number of turns in the actual machine. Let  $R$  be the number of actual turns divided by the number of simulation turns. By using the rms growth rates  $\alpha_{p1} = R\alpha_{p0}$  and  $\alpha_{\perp 1} = R\alpha_{\perp 0}$ , the simulation will show the same growth with  $R$  fewer computations than a direct simulation. The final modification is due to the fact that the line densities in Figure 1 are not close to Gaussian, while equations 1, 2 and 3 are defined for Gaussian bunches. Define a form factor  $F(t) = I(t)\sigma_t 2\sqrt{\pi}/Q$  where  $I(t)$  is the instantaneous beam current,  $\sigma_t$  is the rms bunch length, and  $Q$  is the total bunch charge. The IBS momentum kick given to a particle on a given turn is  $\Delta p = \sigma_p \sqrt{\alpha_{p1} T_0 F(t)} \chi$ , where  $\chi$  is a random deviate with zero mean and unit standard deviation. The rms value of  $\Delta p$  for Gaussian  $I(t)$  equals Piwinski's value. The same form factor is used for transverse kicks.

The effect of longitudinal cooling is shown in Figures 2, 3 and 4. We assume the kick on a given bunch passage is periodic at 5 ns and use FFTs of the line density and transfer impedance to increase computational speed. For the simulation in Figure 2 the ratio of the physical time to the simulation time satisfied  $R = N_{phys}/N_{macro}$  where  $N_{phys}$  was the number of particles in the physical (*ie.* real) beam, and  $N_{macro}$  was the number of macro particles used in the simulation. The reason for this is discussed at length in [22]. The basic idea is that the stochastic cooling rate scales like  $1/N$  where  $N$  is the number of particles. Now imagine that one does a multiparticle simulation like those used for beam stability calculations [24]. For an appropriate definition of gain one can increase  $N_{macro}$  by a factor, say  $x$ , and the cooling time will increase by that same factor of  $x$ . Therefore, scaling the IBS rate by the same factor as the stochastic cooling rate should result in rapid convergence as the number of macroparticles is increased. There are two caveats to this argument. The first is that IBS contributes to the mixing, which improves stochastic cooling [4]. For the simulations presented here the mixing due to IBS is much smaller than mixing due to the RF. Also, one needs to be careful of transient effects associated

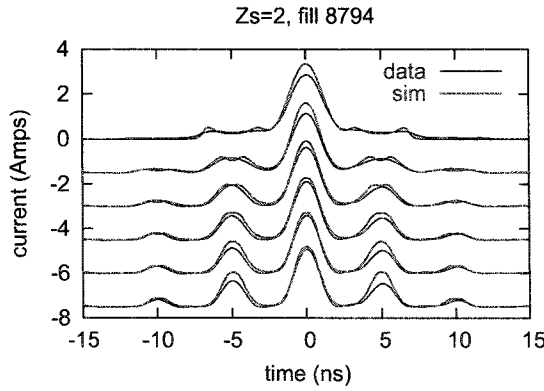


Figure 2: Evolution of a five hour RHIC store with gold beam and good longitudinal cooling. The red lines are wall current monitor data and the blue lines are from a simulation. The initial conditions are shown at the top and the traces are one hour apart. The measured signal suppression in the actual beam was about 6dB, and agrees well with the signal suppression calculated using the simulation. The simulation used 50,000 macroparticles.

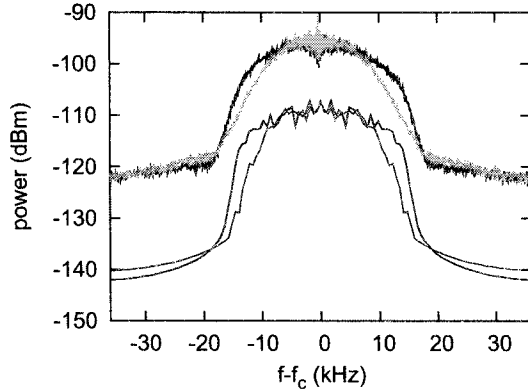


Figure 3: Measured and simulated signal suppression at 6 GHz. The data are the top two traces and the simulation the bottom 2.

with turning on the system, which can result in small but measureable emittance growth.

Since the simulation code includes all dynamical effects, signal suppression is automatically included. A narrow band pickup signal is created by defining a central frequency,  $f_c$  and accumulating the complex numbers  $S_n = \int I_n(t) \exp(2\pi f_c t) dt$ , where  $n$  denotes turn number. Taking the discrete fourier transforms of the real and imaginary parts and summing the squares gives a symmetrized spectrum. Averaging this spectrum over disjoint subsets gives an estimate of the average spectrum. Figure 3 compares data and simulation for the gain used to create Figure 2.

Figure 4 shows the beam current for the cooled (yellow) and uncooled (blue) beams over several stores. With cooling on the yellow beam has no measureable debunching. With the excellent agreement of simulation and data in Fig-

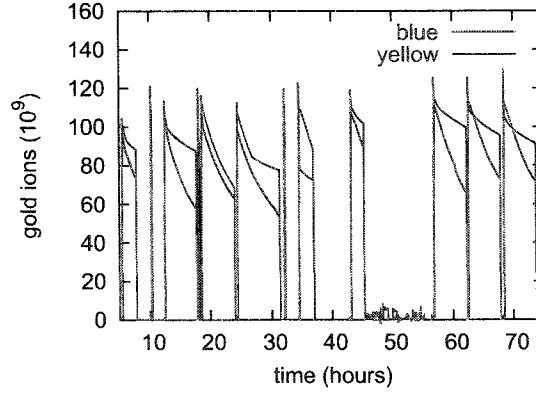


Figure 4: Beam current in RHIC. The store starting at 19 hours had no cooling and the following store had cooling for the second half. All other stores had cooling in yellow from the beginning.

ures 1 through 3, we assert that the code is good enough for design work. We go on to predict beam behavior when transverse cooling is included.

## TRANSVERSE COOLING SYSTEM

Including transverse cooling in the simulation code requires a subroutine to accumulate the dipole density at the pickup location and to apply the derived kick at the kicker location. We assume the same 200 MHz cavity spacing so the kick is periodic at 5 ns for a given bunch passage, just like the longitudinal one. As a starting point we simulated transverse cooling without longitudinal cooling or intrabeam scattering. This parameter regime allows for a particularly clean test of the scaling law for cooling rate as a function of macro-particle number, as shown in Figure 5. The horizontal scale is the normalized longitudinal energy,

$$H_s(\epsilon, \tau) = \frac{T_0 \eta}{2\beta^2 E_0} \epsilon^2 - \int_0^\tau dt q V_{rf}(t), \quad (4)$$

where  $\eta$  is the frequency slip factor,  $E_0$  is the synchronous energy,  $\epsilon = E - E_0$  is energy deviation,  $\tau$  is the arrival time with respect to the synchronous particle, and  $V_{rf}(t)$  is the RF voltage. It would be hard work to prove a statistically significant difference between  $8 \times 10^3$  and  $2 \times 10^6$  macroparticles.

The strong dependence of transverse cooling rate on longitudinal energy was predicted by Chattopadhyay [6, 7], and design options for transverse cooling in the SPS included a higher harmonic RF cavity in an attempt to fix the problem. In RHIC this problem is solved by longitudinal diffusion, from both IBS and the longitudinal stochastic cooling system. Diffusion causes the longitudinal energy of individual particles to migrate, and for RHIC parameters the net effect is a transverse cooling rate that is nearly flat in longitudinal energy. Figures 6 and 7 show simulation results including all of these effects. The simulations

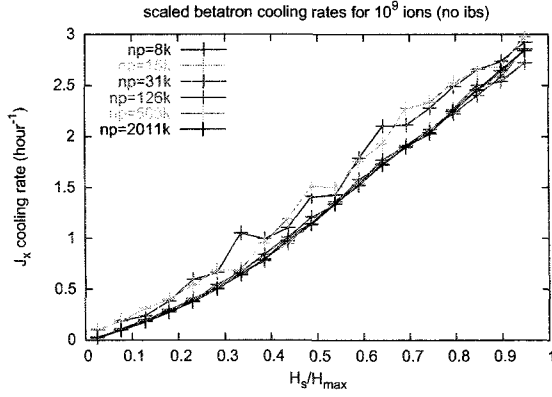


Figure 5: Transverse cooling rate versus the value of the longitudinal hamiltonian. Similar results are shown in [6, 7]

used a slightly larger longitudinal gain than used now for operations.

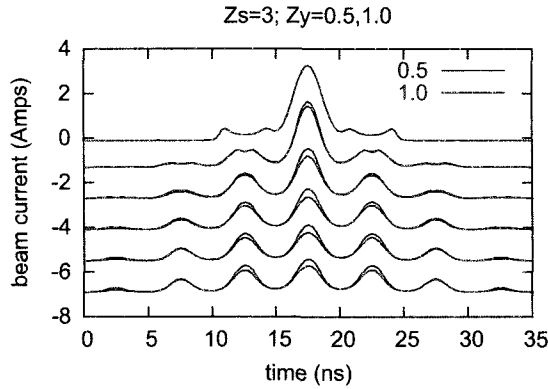


Figure 6: Simulated longitudinal profiles over 5 hours with two different transverse cooling gains.

The simulations in Figures 6 and 7 used the 2/3rd turn delay we have in the yellow ring. For the blue (clockwise) ring we are building a system that uses a 70 GHz microwave link that allows for 1/6th turn delay. Additionally, we hope to generate 5 MV on the  $h = 2520$  RF system and to get clean rebucketing. Figures 8 and 9 give an indication of our options if these goals are achieved.

We envision a transverse cooling system that looks very much like our longitudinal cooling system. Cavities capable of producing transverse kicks between 5 and 8 GHz are straightforward to build. Define a horizontal voltage as

$$V_x = \int ds (E_x + cB_y) e^{2\pi i \omega_r s / c},$$

where  $\omega_r$  is the cavity resonant frequency. Define the horizontal impedance through  $P = V_x^2 / 2R_x$ , with  $P$  the input power. Two cell cavities can develop  $R_x / Q \approx 20\Omega$ . For Figures 6 through 9 the rms transverse kick satisfied  $\langle V_x^2 \rangle^{1/2} \leq 250$  V at the average  $\beta$  function of 21 m.

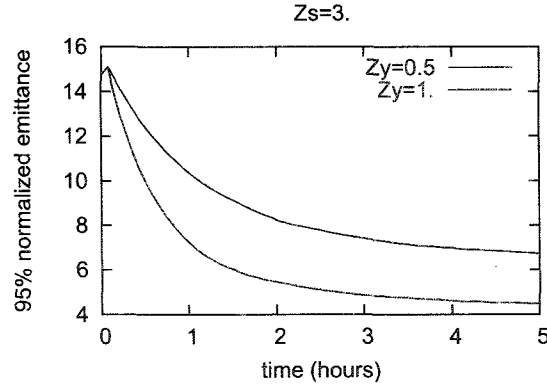


Figure 7: Simulated transverse emittance over 5 hours with two different transverse cooling gains.

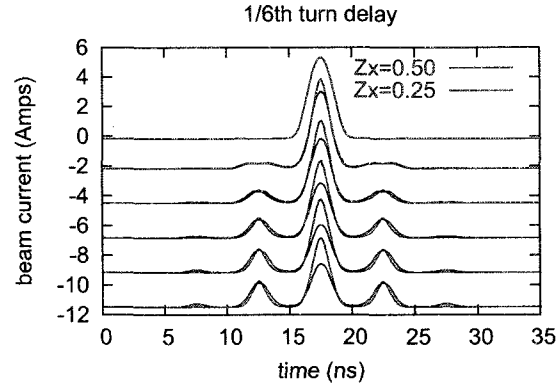


Figure 8: Simulated longitudinal profiles over 5 hours with two different transverse cooling gains and 1/6th turn delay. The transverse gain of 0.25 utilized only a single one turn delay in the longitudinal cooling system, while the gain of 0.5 used the same cascaded delays we use now.

With 16 cavities this corresponds to about 60 V/cavity. At 5 GHz,  $Q \approx 500$  and the rms power is  $P = \langle V_x^2 \rangle / R_x = 0.4$  W. Allowing for 3 dB of attenuation and a  $3\sigma$  voltage yields an amplifier power of 7 Watts. Detailed pickup design is only beginning, and we are leaning toward slotted waveguides [28].

Low level signal processing for a transverse cooling system in RHIC must deal with oscillations of the closed orbit due to mechanical vibrations of the triplet quadrupoles [25, 26]. Oscillation amplitudes 5 to 10% of the rms beam size with frequencies of order 10 Hz are typical. These oscillations will cause the transverse Schottky signal to be polluted by the coherent lines of the longitudinal signal and, to a lesser extent, the longitudinal Schottky signal. We will use an optical notch filter to suppress these signals [27]. The system is only a slight development from what we have now.

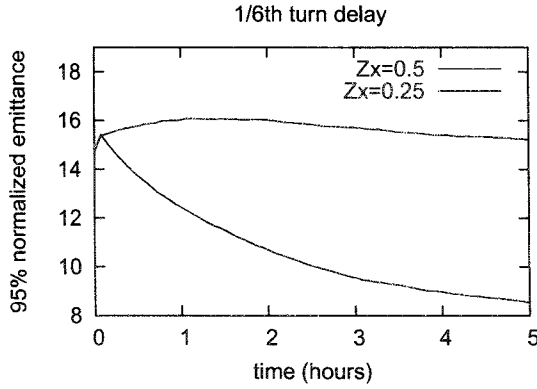


Figure 9: Simulated transverse emittance over 5 hours with two different transverse cooling gains. The parameters are the same as those in Fig 8

## DISCUSSION AND CONCLUSIONS

In both the data and simulations the longitudinal beam profile develops unwanted satellite bunches, which reduce the useful luminosity. To shed light on this phenomena consider the diffusion equation used to study quantum lifetime in electron storage rings [29],

$$\frac{\partial F(\epsilon, \tau, n)}{\partial n} + \frac{\partial H_s}{\partial \epsilon} \frac{\partial F}{\partial \tau} - \frac{\partial H_s}{\partial \tau} \frac{\partial F}{\partial \epsilon} = \frac{\partial}{\partial \epsilon} \left( \chi \epsilon F + \alpha_\epsilon \frac{\partial F}{\partial \epsilon} \right), \quad (5)$$

where turn number  $n$  is the time-like variable,  $\chi$  is the cooling rate, and  $\alpha_\epsilon$  creates emittance growth. Equation (5) has a unique time independent solution,  $F(\epsilon, \tau) = C_0 \exp(-H_s(\epsilon, \tau)/H_0)$ , where  $C_0$  is a normalization constant and  $H_0 = \alpha_\epsilon T_0 \eta / \chi \beta^2 E_0$ . If we take this solution seriously, the only way to keep satellite bunches small is to reduce  $H_0$ , or change the RF voltage. In particular, the potential difference between the satellite buckets and the main bucket must be larger than  $H_0$ . For beams dominated by IBS,  $\alpha_\epsilon$  increases as  $C_0$  increases. For stochastic cooling,  $\chi$  decreases as  $C_0$  increases. These opposing forces make it difficult to reduce  $H_0$  and tighten the beam. Future work will address this issue more thoroughly, but for the present we can draw some conclusions.

Stochastic cooling for colliding beams, with RHIC particle densities, is now a proven technology. The systems are inexpensive by collider standards and many parts can be bought off the shelf.

## ACKNOWLEDGEMENTS

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